ME-221
PROBLEM SET 7

Problem 1

Calculate $x(t)$ corresponding to the following Laplace transforms:

a) $X(s) = \frac{s(s+1)}{(s+2)(s+3)(s+4)}$

b) $X(s) = \frac{s+4}{(s+1)^2}$

c) $X(s) = \frac{2}{(s+1)^2(s+2)}$

d) $X(s) = \frac{5e^{-2s}}{s^3+2s^2+5s}$

Problem 2

Calculate the output $y(t)$ of the following system for the given input signals.

$$\ddot{y}(t) + 7\dot{y}(t) + 6y = u(t) \qquad y(0) = 1, \quad \dot{y}(0) = 2$$

(a) The input is given as a unit step function. Start by separately calculating the forced and natural responses, then sum them to obtain the total response.

(b) An input pulse is applied which is defined as $u(t) = 1$ for $0 \leq t < 5$ and $u(t) = 0$ otherwise. Hint: Use the result you found in (a).

Problem 3

Solve the following differential equation using the Laplace transform.

$$\ddot{x}(t) + 2\dot{x}(t) + x(t) = -e^{-t}sint \qquad x(0) = 0, \quad \dot{x}(0) = 2$$

Problem 4

We are given a system with the transfer function $G(s) = \frac{2}{(s+3)^2}$

(a) If the input is given by $u(t) = \epsilon(t)e^{-2t}$ then calculate the output $y(t)$ of the system.

(b) Use the convolution operation to calculate $y(t)$ for the same input given in part (a).

(c) Derive a state-space representation for this system.